University of Waterloo

CO370 Fall 2025

Deterministic OR Models

Final Report

Optimizing Bus Schedules to Minimize Total Waiting Time Across GO Transit System

Group 6
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February 16, 2026

Abstract

As students in Ontario who use GO Transit to travel between cities, we often experience highly varying transfer waiting time on the GO Transit network. At different GO Stations, connecting times can range from a few minutes to nearly two hours, making trips longer and less reliable. This project aims to improve schedule coordination across transfer stations to reduce total passenger transfer waiting time without requiring new infrastructure.

Our model indicates that, given our assumptions and allowing feasible and realistic schedule adjustments, the current GO transit schedule still exist room for optimizing. By rescheduling arrivals and departures within realistic limits while enforcing capacity and minimum transfer time requirements, the optimized schedule can reduce total passenger transfer waiting time during peak commute periods. This suggests that improved schedule alignment, without adding new services or infrastructure could enhance the reliability and consistency of the Go Transit network.

1 Introduction

In Ontario, as in many regions with substantial city-to-city commuting, public transportation plays an important role in people's daily lives. The GO Transit network offers affordable and convenient commute options for students, workers, and other riders. However, despite the broad reach of GO buses and trains, there is still significant potential for the network to improve efficiency. Transfer experiences at GO stations are a particular concern, as many passengers face long and unpredictable waiting times. Sometimes a connecting bus or train arrives within minutes, while at other times, riders may wait more than two hours. Such uncertainty increases total travel times, undermines system reliability, and increases stress for passengers who rely heavily on public transit.

Based on the current GO Transit schedules, we expect our model to output the greatest improvements at busy transfer stations where multiple routes intersect and existing transfer times are particularly long. Also, we expect that the optimal schedule produced by our model will reduce transfer waiting time and provide a more consistent and reliable experience for GO Transit passengers during peak hours.

Through our project, we aim to minimize the total transfer waiting time for all GO Transit passengers. We consider a network of 35 main transfer stations across the GTA and focus on passengers making connections at these stations. Our Mixed Integer Programming (MIP) model adjusts the schedule by allowing each trip to be shifted slightly earlier or later. Binary connection variables determine whether an arrival trip connects to a given departure, while capacity constraints ensure that the number of passengers assigned to each departure does not exceed available seating. We also enforce a minimum transfer time at each hub so that passengers have enough time to transfer to the connecting trip, and we include a penalty cost for missed connections to prevent model from leaving riders behind. The model is built using official GO Transit GTFS schedule data, combined with reasonable assumptions about the number of passengers transferring between individual routes.

2 Assumption

In order to have a solvable model, we had to make some assumptions about the GO Transit system we are modeling. Many of these assumptions are, to the best of our knowledge, realistic approximations that do not take away from the relevance of our results. For example, one assumption is that that passengers value their waiting time equally, so each passenger minute of waiting is weighted the same in the objective function. We assume that the minimum transfer time M_h at each hub is sufficient for passengers to walk between platforms and board the next vehicle, based on station layouts and walking distances.

We also assume that the maximum wait time v_{ijh} is chosen large enough to cover all connections, but small enough to avoid extremely long transfers that most passengers would not accept. In addition, we assume that once a connection is chosen, all passengers assigned to that connection will board the departure, up to the available capacity, and that there are no additional priority rules at each platform.

On the other hand, some of our assumptions simplify the GO transit network but were necessary to keep the model tractable. We don't consider cancellations and extreme delays caused by weather conditions. And we do not allow the model to add entirely new trips or remove existing ones. Also, route frequencies must stay within a reasonable range, and our schedule adjustments only shift trips slightly earlier or later. This allows us to focus on improving the planned schedule rather than unpredictable day-to-day variation.

We restrict our model to focus on weekday rush hours from 6–10 AM and 4–8 PM as many GO passengers use the system to commute to work or school, so transfer time during rush hour is especially important and it affects the largest number of passengers.

Finally, we have made some assumptions about data since we only have a limited access to the official GO transit data but that could be refined if more detailed information were available. GO Transit does not publish the exact number of passengers transferring between every pair of routes and stations, so we need to estimate the transfer volumes D_{ijh} using publicly available schedule information and reasonable assumptions about passenger behavior. We also exclude budget constraints from the model and assume that implementing minor schedule changes would not significantly increase operating costs.

3 Model

3.1 Variables

- $Z_{ijh} \in \{0,1\}$: 1 if arrival trip i connects to departure trip j at hub h, 0 otherwise.
- $W_{ijh} \geq 0$: counted waiting time (in minutes) for passengers transferring from trip i to trip j at hub h.
- $d_i \geq 0$: schedule delay(in minutes) applied to trip i (move later).
- $a_i \geq 0$: schedule advance(in minutes) applied to trip i (move earlier).
- $y_{ijh}(free)$: The physical time difference between the adjusted arrival of trip i and the adjusted departure of trip j at hub h.

3.2 Constants & Set

- $T = \{t_1, t_2, \ldots, \}$: a set of all trips.
- $H = \{h_1, h_2, \ldots, h_{35}\}$: 35 main transfer hubs.
- M_h: Minimum required transfer time at hub h.
- S_i^A: Scheduled arrival time of trip i.
- S_j^D: Scheduled departure time of trip j.
- D_{ijh}: Estimated number of passengers transferring from i to j at hub h.
- C_{jh}: Passenger capacity for departure trip j at hub h.
- k_1: Maximum allowable schedule advance for trip t.
- k_2: Maximum allowable schedule delay for trip t.
- v_{ijh}: Maximum wait time from i to j at hub h.
- $L = 300$: a large enough number that is greater than the actual waiting time.
- $P = 60$: Penalty cost for a missed connection.

3.3 Objective Function

$$\text{Min} \quad \sum_{(i,j)\in T} \sum_{h\in H} D_{ijh} \cdot (W_{ijh} + P \cdot (1 - Z_{ijh}))$$

3.4 Constraints

$$\begin{aligned}
&(1) \quad a_t \le k_1, d_t \le k_2, && \forall t \in T \\
&(2) \quad \sum_i D_{ijh} \cdot Z_{ijh} \le C_{jh}, && \forall (j \in T, h \in H) \\
&(3) \quad y_{ijh} = (S_j^D + d_j - a_j) - (S_i^A + d_i - a_i), && \forall (i, j \in T, h \in H) \\
&(4a) \quad y_{ijh} \ge M_h \cdot Z_{ijh} && \forall (i, j \in T, h \in H) \\
&(4b) \quad y_{ijh} \le v_{ijh} \cdot Z_{ijh} + L(1 - Z_{ijh}), && \forall (i, j \in T, h \in H) \\
&(5) \quad W_{ijh} \ge y_{ijh} - L(1 - Z_{ijh}), && \forall (i, j \in T, h \in H) \\
&(6) \quad W_{ijh} \le y_{ijh} + L(1 - Z_{ijh}), && \forall (i, j \in T, h \in H) \\
&(7) \quad W_{ijh} \le L \cdot Z_{ijh}, && \forall (i, j \in T, h \in H) \\
&(8) \quad Z_{ijh} \in \{0, 1\}, \quad d_i, a_i, W_{ijh} \ge 0, && \forall (i, j \in T, h \in H)
\end{aligned}$$

3.5 Explanation

- **Constraint (1)**: places upper bounds on how much we are allowed to adjust the scheduled time of each trip. We do not allow trip to be moved earlier by more than k_1 minutes and we do not allow trip to be moved later by more than k_2 minutes.

- **Constraint (2)**: ensures the total number of passengers transferring to departure trip j at hub h from all arriving trip i cannot exceed the departure bus/train capacity.

- **Constraint (3)**: defines the actual waiting time y_{ijh} after adjustments between the adjusted departure of trip j and the adjusted arrival of trip i at hub h. The net shift for a trip i is given by $d_i - a_i$, if the net shift is negative means we shift the trip earlier, if it's positive means we shift the trip later.

- **Constraint (4)**: (a) enforces the minimum transfer time M_h only when $Z_{ijh} = 1$, and when $Z_{ijh} = 0$, (b) enforces the specific waiting time limits defined by u_{ijh} and v_{ijh} based on the connection status.

- **Constraint (5)**: links W_{ijh} to y_{ijh}. When $Z_{ijh} = 1$, the constraint becomes $W_{ijh} \geq y_{ijh}$, ensuring that the waiting time is at least the actual adjusted transfer gap. When $Z_{ijh} = 0$, we have $y_{ijh} - L$ and allowing W_{ijh} to be zero for connections that are not chosen.

- **Constraint (6)**: provides the upper bound between W_{ijh} and y_{ijh} using the same L. When $Z_{ijh} = 1$, the constraints become $W_{ijh} \leq y_{ijh}$ and this together with Constraint (5) forces $W_{ijh} = y_{ijh}$ for selected connections. When $Z_{ijh} = 0$, we have $y_{ijh} + L$ and allowing W_{ijh} to be zero for connections that are not chosen.

- **Constraint (7)**: sets an upper bound on W_{ijh}. When $Z_{ijh} = 1$, the constraint becomes $W_{ijh} \leq L$, which is a safe bound since L is selected to be large enough than any feasible waiting time. When $Z_{ijh} = 0$, the constraint forces $W_{ijh} \leq 0$, so together with $W_{ijh} \geq 0$ it ensures $W_{ijh} = 0$ for connections that are not chosen.

- **Constraint (8)**: defines binary and non-negativity restrictions.

4 Data

4.1 Filter

The data for this study comes from GO Transit's public GTFS feed, which includes over 72,000 trips across all days and modes. To keep things grounded in a realistic operating pattern, we focused on a single representative weekday. That gave us 1,827 trips running on that day. From there, we manually selected 164 trips that are especially relevant for transfers at key hubs. These trips formed 116 potential connections, defined as "arrival trip $\rightarrow$ departure trip" at the same location. This manual step helped us narrow the scope to only meaningful connections, the ones that people are likely to actually use rather than trying to optimize across all possible combinations.

Once the set of trips was selected, we wrote scripts to clean and structure the data so it could be used by the optimization model. We pulled out unique trip IDs and matched them with their route names using GTFS route fields like route_id and route_short_name, which helped us identify vehicle types and service lines. Arrival and departure times from stop_times.txt were converted into a continuous time scale in minutes past midnight (fields like arr_time_min and dep_time_min), so the model could calculate wait times easily.

We also assigned vehicle capacities using published specs: 53 seats for single-deck buses, 81 for double-deckers, and 1,944 for GO trains. These limits were attached to each departure trip and enforced during optimization, ensuring that no vehicle ends up with more transferring passengers than it can handle.

4.2 Estimation

To implement the optimization model, we required specific operational details that are not provided in standard public schedule data. Because we lack data that tracks individual passenger movements between vehicles, we derived estimates based on existing service schedules and network constraints. The following subsections describe how we determined the values for four key components: vehicle passenger capacity (C_{jh}), minimum transfer time buffers (M_h), and passenger demand (D_{ijh}).

4.2.1 Passenger capacity of vehicle for departure trip j at hub h(C_{jh})

To effectively model capacity constraints (Constraint 2), we incorporated a heterogeneous fleet structure derived from GO Transit's rolling stock specifications. Each trip j in the `trips.csv` dataset was assigned a deterministic capacity parameter C_{jh} corresponding to one of three distinct vehicle classes:

- **Single-Deck Buses ($C_{jh} = 53$):** Representing the baseline capacity for regional bus routes (e.g., Route 68, 47) operating standard 45-foot highway coaches (MCI D4500).
- **Double-Deck Buses ($C_{jh} = 81$):** Assigned to high-demand routes where ADL Enviro500 SuperLo or similar double-deck vehicles are deployed to alleviate peak-hour load. This higher capacity limit mitigates the risk of false infeasibility on popular routes (e.g., Route 25c, 30).
- **GO Trains ($C_{jh} = 1944$):** Denoting the maximum theoretical capacity of a standard 12-coaches Bombardier BiLevel trainset. These values are applied to all trips designated with rail identifiers (e.g., KI, LW, BR, ST lines) to accommodate high-volume inter-regional transfer pulses.

4.2.2 Minimum Transfer Time Calibration (M_h)

The parameter M_h establishes a lower bound on the slack variable y_{ijh} (Constraint 4a), representing the non-negotiable buffer required for passenger transfer at hub h. To mitigate the risk of generating infeasible schedules, we eschewed a uniform global parameter in favor of a topology-dependent classification system derived from the `connections.csv` dataset.

- **Tier 1: Major Transfer Hubs** ($M_h = 10$ minutes): This classification applies to 38 connections within the dataset, specifically at major hubs such as Union Station (UN), Kitchener GO, Aldershot, and Kipling. These hubs are defined by intermodal transitions (e.g., Train-to-Bus) and high passenger density, necessitating a conservative buffer to ensure transfer feasibility.
- **Tier 2: Standard Transfer Hubs** ($M_h = 5$ minutes): There are 77 connections in this category. Hubs such as Erin Mills, McMaster University and Burlington are featuring bus-to-bus transfers. A 5-minute window provides a sufficient transfer buffer for passengers.
- **Tier 3: Rapid Transfer Hubs** ($M_h = 4$ minutes): A single distinct transfer hub in the dataset, Trafalgar Rd.@ Hwy 407, was assigned a minimal buffer of 4 minutes. This reflects a scenario with negligible transfer time, likely no other transfer are supported at this connection or the hub is too small, so friction of transfer is minimal.

4.2.3 Estimated number of passengers transferring from i to j at hub h (D_{ijh})

Since detailed ridership data are not publicly available, the estimates combine observed aggregate data (such as station boardings, annual ridership and population figures) with a small set of modeling assumptions. The goal is to obtain consistent and reasonable values suitable for timetable analysis, rather than precise forecasts.

- **Traveling Numbers for Selected Trip**:

  Let Q_{st} denote the estimated number of passengers traveling from origin s to destination t per typical weekday during peak hours in 2025. For each pair (s,t), we first construct a preliminary estimate Q_{st}.

  We begin by defining r_s as an estimate of the total number of travelers departing from origin s per day. Depending on the context, r_s may be derived from station-level GO "home rider" counts, annual GO reports, or other transit boarding statistics converted to a daily average. Where necessary, outdated data values are scaled to represent the amount in 2025 using growth factors, reflecting partial ridership recovery. The growth factors are retrieved from comparing past records.

  Given an origin s, we identify a set of destinations $t \in \mathrm{Des}_s$ that we want to model (for instance, Niagara Falls, McMaster University, Mount Joy GO and Maple GO for trips starting at the University of Waterloo). For each destination t, we associate a population pop_t. (the region population, for instance), and define a population-based share. In the simplest case, where we distribute r_s across the destinations in Des_s, we use

$$\theta_{st} = \frac{\mathrm{pop}_t}{\sum_{t \in \mathrm{Des}_s} \mathrm{pop}_t},$$

  so that $\sum_{t \in \mathrm{Des}_s} \theta_{st} = 1$. The demand is then given by

$$Q_{st} = r_s \, \theta_{st} .$$

In other situations, where destination t is compared against a larger region including it and other potential destinations, the denominator in the definition of θ_{st} is taken to be the destination regional total. In all cases, θ_{st} is interpreted as the share of travelers from s who head to the destination t.

The demand for a selected full trip from s to t is then defined by

$$Q_{st} = r_s \, \theta_{st} \, .$$

- **Itinerary Allocation**:

  For each pair (s,t), the total demand Q_{st} is distributed across the feasible GO itineraries connecting s and t. Let these itineraries be indexed by $k = 1, \ldots, n$.

  For each itinerary k, we compute the total travel time T_{stk} (in minutes) from the initial departure at origin s to the final arrival at destination t, including transfer times. We assume that passengers prefer shorter itineraries, and we represent this with an inverse-time weighting:

  $$w_{stk} = \frac{1}{T_{stk}} \, .$$

  These weights are normalized to obtain route choice probabilities

  $$p_{stk} = \frac{w_{stk}}{\sum_{k=1}^{n_{st}} w_{stk}} \, , \quad k = 1, \ldots, n \, ,$$

  so that $\sum_{k=1}^{n} p_{stk} = 1$ for each pair (s,t).

  The expected number of passengers who select itinerary k for the pair (s,t) is then

  $$N_{stk} = p_{stk} \, Q_{st} \, .$$

  By construction, the sum of the itinerary flows recovers the total demand:

  $$\sum_{k=1}^{n} N_{stk} = Q_{st} \, .$$

- **Hub Flows**:

  In the network formulation, we are interested in how many passengers transfer at specific hubs. Let h denote a transfer hub (for example, Union Station, Square One, or Hwy 407 Bus Terminal). For each pair of ordered nodes (i,j) and hub h, we define D_{ijh} as the estimated number of passengers transferring from i to j at hub h. By definition, D_{ijh} is a hub-flow quantity, not a total trip demand.

  For example, if a path for some pair (s,t) contains a part from i to j that passes through the hub h, then the corresponding value D_{ijh} can be larger than or equal to Q_{st}, because D_{ijh} aggregates all passengers traveling from i to j via h, possibly belonging to multiple (s,t) pairs. Moreover, some passengers may transfer from i to j at hub h and subsequently continue to destinations other than t.

  In practice, we obtain the values D_{ijh} by scaling demands Q_{st} along the chosen routes, with minor adjustments made to ensure consistency in the simulation model.

4.3 Trips Study

To validate the dataset, we analyze the critical itinerary linking the **University of Waterloo** to **Niagara Falls**. This route represents a primary desire line for students traveling between a major hub and the Niagara region.
Route (example 06:20 departure): University of Waterloo (06:20) → Erin Mills (Arr 07:50 - Dep 07:55) → Dundas @ Hwy 407 (Arr 08:23 / Dep 08:42) → Niagara Falls (10:27).

Transfer Hub	Itinerary Schedule (segment context)
1. Erin Mills (transfer)	**Incoming leg:** University of Waterloo → Erin Mills Route 25 (**Arr** 07:50) **Outgoing leg:** Erin Mills → Dundas @ Hwy 407 Route 47G (**Dep** 07:55)
2. Dundas @ Hwy 407 (transfer)	**Incoming leg:** Erin Mills → Dundas @ Hwy 407 Route 47G (**Arr** 08:23) **Outgoing leg:** Dundas @ Hwy 407 → Niagara Falls Route 12 (**Dep** 08:42) *(Final arr. Niagara Falls 10:27)*

Table 1: Waterloo → Niagara Falls itinerary with transfers at Erin Mills and Dundas @ Hwy 407

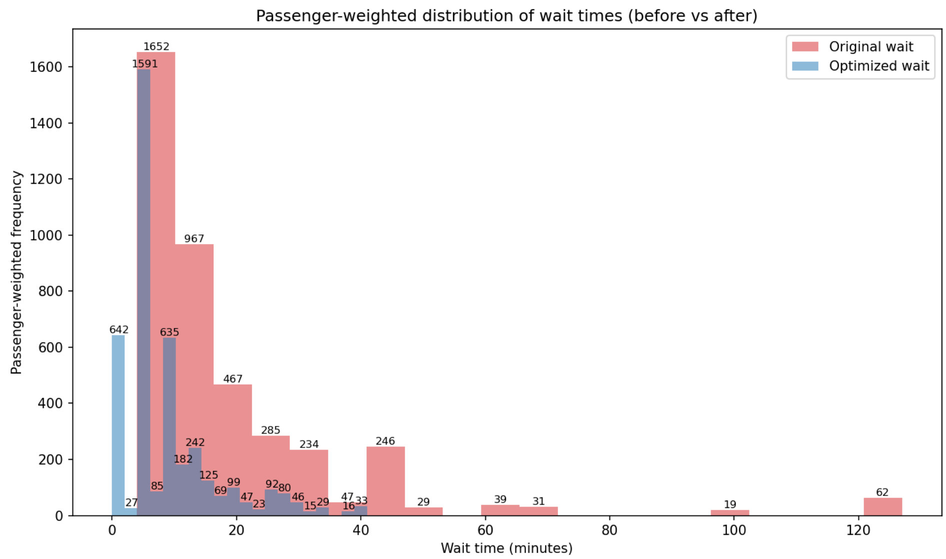
Optimization Analysis.

- **Erin Mills (Transfer 1):** Original gap = 5 min. Action: *maintained*. The model locks this tight connection ($M_h = 5$) to reduce miss risk for 47 passengers.
- **Dundas @ Hwy 407 (Transfer 2):** Gap reduced 19 → 5 min. Logic: delay arrival by 10 min ($d_i = 10$) and advance departure by 4 min ($a_j = 4$). Math: $19 - 10 - 4 = \mathbf{5}$.

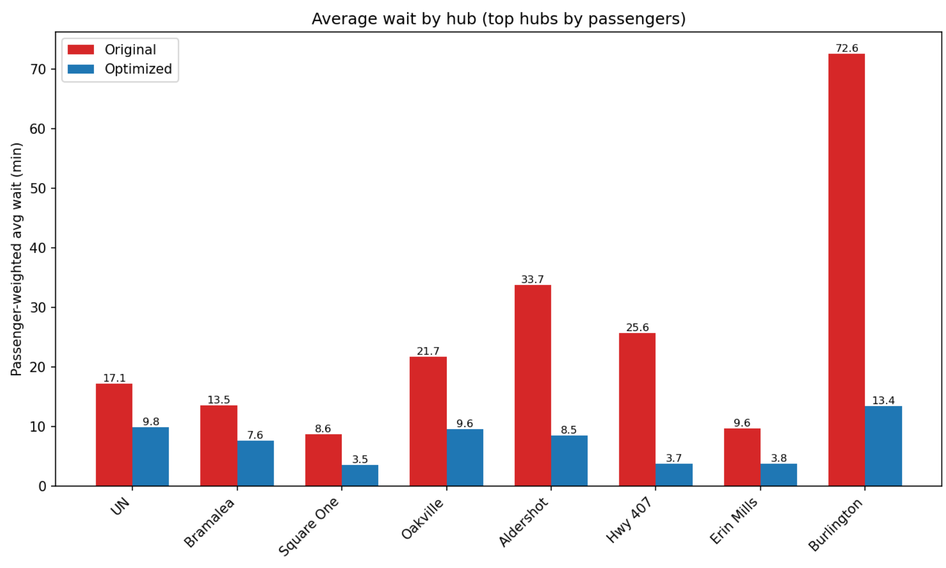
5 Results

We present aggregate results for the network and provide detailed results for each connection in the appendix. To summarize performance, we report passenger-minutes of waiting and passenger-weighted average wait time. Passenger-minutes of waiting are defined as the total waiting minutes, summed over all passengers (for example, 10 passengers each waiting 5 minutes corresponds to 50 passenger-minutes in total). The passenger-weighted average wait time is the average wait that a typical passenger experiences, computed by weighting each connection's total wait time divided by the number of passengers using that connection.

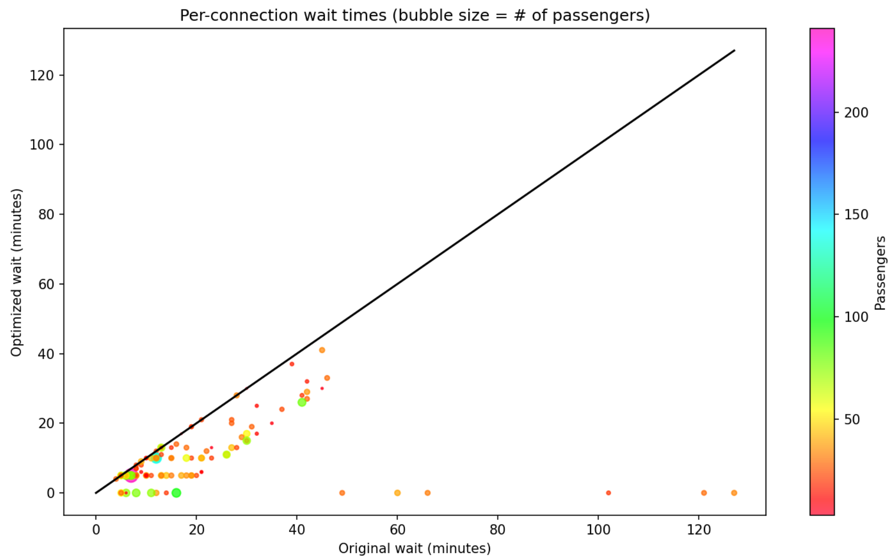
In total, the schedule adjustments cut system-wide transfer waiting time by about 52%, from 75,425 to 36,113 minutes for all passengers.



The histogram of passenger-weighted wait times before and after optimization shows a clear compression in the distribution. Prior to optimization, wait times were widely dispersed, with a significant portion of passengers experiencing waits between 15 to 40 minutes and some connections even reaching above 100 minutes. After optimization, the bulk of passengers were shifted into much shorter wait ranges, with most concentrated in the 5 to 15-minute window. Long waits were largely eliminated. This demonstrates that the optimization was effective not just in reducing average wait times, but also in improving consistency and fairness for all passenegers.



When examining the top transfer hubs by total passenger volume, we observe significant reductions in average passenger-weighted wait times across nearly all of them. Notably, Union Station (UN), the busiest hub by far, saw average wait times drop from approximately 17.1 to 9.8 minutes, resulting in a savings of over 8,300 passenger-minutes. Similarly, Bramalea, another high-volume hub, experienced a reduction from 13.5 to 7.6 minutes, saving more than 2,450 passenger-minutes. These gains are particularly impactful because they affect a large number of passengers, maximizing the system-wide benefit of the schedule adjustments.



The scatter plot comparing each connection's original wait time versus the optimized wait time provides a clear visualization of the effectiveness of the optimization. Each point represents a transfer connection, with the bubble size proportional to the number of passengers using that connection. The vast majority of points lie below the diagonal line, indicating that the optimized wait time is shorter than the original. Importantly, the largest bubbles which representing high-demand transfers, also tend to lie below the line, confirming that our optimization model prioritized improvements where they would benefit the most riders. No connections moved above the diagonal, meaning that no transfer was made worse, preserving overall service quality while achieving substantial efficiency gains.

6 Limitations

Our model is built on a simplified version of the GO Transit network and focuses specifically on weekday rush hours (6–10 AM and 4–8 PM). As such, the results reflect improvements to a typical commuter schedule and may not fully capture off-peak, weekend, or special-event service patterns. The optimization is limited to making small adjustments to the existing timetable. It does not consider adding or removing trips, major frequency changes, or broader network redesigns.

We also rely on estimated transfer volumes and simplified assumptions about passenger behavior, such as assigning equal weight to each passenger-minute of waiting. Due to limited access to detailed operational and demand data, our final results should be interpreted with caution. Still, the relative improvements and patterns we observe offer useful insights into where schedule adjustments can provide the greatest benefit. With more detailed data, the model could be extended to account for additional real-world constraints and priorities.

7 References

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Appendix

All code, raw data, and full optimization results referenced in this report are available in our GitHub repository:

`github.com/ru1ru1ru1ru1/CO370-F25-Project-Group6`

This includes:

- Full connection-level results (before and after optimization)
- Source code for the optimization model
- Data preprocessing scripts
- Visualization code and generated figures